

Two-loop β -functions for the baryon number violating scenario of the MSSM

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ABSTRACT: We present the full two-loop β -functions for the minimal supersymmetric standard model, extended to include baryon number violating couplings through explicit R -parity violation. We also consider the effect of two-loop running on the sparticle spectrum.

KEYWORDS: Supersymmetric Standard Model, Supersymmetry Phenomenology.

Contents

1. Introduction	1
2. The R-parity violation scenario	2
2.1 The baryon number violation scenario of the MSSM	3
3. Renormalization group equations and the soft β-functions	3
3.1 β -functions	3
4. The numerical calculation of the sparticle masses	10
5. Conclusions	16

1. Introduction

The possible appearance of R -parity violating couplings, and hence the question of lepton and baryon number non-conservation, has been emphasized since the early development of supersymmetric theories. The rich phenomenology implied by R -parity violation has now gained full attention in the search for supersymmetry. The R -parity violating terms in supersymmetry have theoretical and phenomenological implications in supersymmetric theories. These new terms can give mass to the neutrino, and also may solve experimental observed discrepancies [1].

In the Standard Model(SM), the baryon (B) and lepton (L) number are conserved automatically because it is impossible to write down renormalizable, gauge-invariant interactions that violate baryon and lepton number. However, in supersymmetric extensions of the SM (where, for each ordinary particle, there is a partner) new interactions are allowed to violate baryon or lepton number. The scale of possible baryon and lepton number violations is associated with the masses of the superpartners responsible for the violations, which may lead to unacceptably large effects. To avoid the interactions which violate baryon and lepton numbers and lead to unacceptably large effects R -parity has been introduced, and it leads to the popular Minimal Supersymmetric Standard Model (MSSM). However, since R -conservation is not theoretically motivated by any known principle, the possibility of R -nonconservation deserves consideration. If we require a supersymmetric Standard Model which is only minimal in particle content the superpotential is modified to allow for additional R -parity violating interactions. R -parity violating interactions come from either superpotential terms which violate baryon number, separate terms which violate lepton-number, or from soft supersymmetry breaking terms. There are different kinds of such terms, of dimensions 4, 3, or 2, with a potentially rich flavour structure.

In the following work we shall write down explicitly all possible R -parity violating terms in the framework of the MSSM, assuming the most general breaking of R -parity. Then we can consider particular scenarios which allow us to reduce the number of independent couplings used to parameterize R -parity violation. The effects of inclusion of R -parity violating couplings have been studied by some authors [2–5] and [6].

In this paper we investigate the effects of violation of the baryon (B) number on the MSSM context. We shall consider the standard minimal supergravity (mSUGRA) scenario including baryon number violating terms. The scenario prohibits the lepton - number violating interactions; however it does not give a discrete gauge anomaly free symmetry [7]. We study some effects of the two-loop renormalization group equations with including all supersymmetric baryon number violation terms. We present explicitly the full two loop beta functions; also we investigate the effects of these new terms on the spectrum of sparticles.

2. The R -parity violation scenario

We begin by reviewing the Lagrangian of the MSSM including R -parity violating extension.

Assuming R -parity invariance, the superpotential of the MSSM Model is given by:

$$W_R = \epsilon_{ab} \left((Y_U)_{ij} Q_i^b U_j^c H_2^a + (Y_D)_{ij} Q_i^b D_j^c H_1^a + (Y_L)_{ij} L_i^b E_j^c H_1^a - \mu H_1^a H_2^b \right). \quad (2.1)$$

In the absence of R -parity, however, R -parity violating terms allowed by renormalizability and gauge invariance must also be included in the superpotential. Therefore, the most general renormalizable, R -parity violating superpotential consistent with the gauge symmetry and field content of the MSSM is given by [8, 9]:

$$W = W_R + \epsilon_{ab} \left(\frac{1}{2} \lambda_{ijk} L_i^a L_j^b E_k^c + \lambda'_{ijk} L_i^a Q_j^{xb} D_{kx}^c \right) + \frac{1}{2} \epsilon_{xyz} \lambda''_{ijk} U_i^{cx} D_j^{cy} D_k^{cz} - \epsilon_{ab} \kappa^i L_i^a H_2^b. \quad (2.2)$$

In equations (2.1) and (2.2), there is a summation over the generation indices ($i, j, k = 1, 2, 3$), and gauge indices ($x, y, z = 1, 2, 3$, and $a, b = 1, 2$). ϵ_{ab} and ϵ_{xyz} are totally anti-symmetric tensors.

In eq. (2.2), the terms proportional to λ'' violate baryon number, whereas the terms proportional to λ , λ' and κ violate lepton number. Because of unobserved proton decay in nature, the simultaneous presence of both types of terms in the superpotential is forbidden [6]. Moreover, cosmological and phenomenological upper bounds do exist on the magnitudes of the λ , λ' [10]. It has been shown cosmological bounds on the λ'' couplings are not strong [11]. In particular, a grand unified theory era leptogenesis makes λ'' components free of cosmological constraints [11]. Therefore, the λ'' components, apart from λ''_{211} and λ''_{311} which have been strongly bounded from the lack of any observed $\bar{n}n$ oscillation, stand relatively unconstrained [6].

In this paper we shall consider the scenario which the superpotential has only been included baryon number violating terms as well as R -parity conserving terms.

2.1 The baryon number violation scenario of the MSSM

The baryon number violating MSSM superpotential is given by:

$$\begin{aligned}
 W = & \epsilon_{ab} \left((Y_U)_{ij} Q_i^b U_j^c H_2^a + (Y_D)_{ij} Q_i^b D_j^c H_1^a + (Y_L)_{ij} L_i^b E_j^c H_1^a - \mu H_1^a H_2^b \right) \\
 & + \frac{1}{2} \epsilon_{xyz} (\Lambda_{U_i})_{jk} U_i^{cx} D_j^{cy} D_k^{cz},
 \end{aligned} \tag{2.3}$$

here, $(\Lambda_{U_i})_{jk} = -(\Lambda_{U_i})_{kj} = (\lambda)''_{ijk}$, and gauge invariance enforces antisymmetry of the λ''_{ijk} couplings with respect to their last two indices.

The soft SUSY braking Lagrangian including the soft baryon number violating terms is give by:

$$\begin{aligned}
 -L_{soft} = & (m_{\tilde{Q}}^2)_{ij} \tilde{Q}_i^\dagger \tilde{Q}_j + (m_{\tilde{L}}^2)_{ij} \tilde{L}_i^\dagger \tilde{L}_j + (m_{\tilde{U}^c}^2)_{ij} \tilde{U}_i^{\dagger c} \tilde{U}_j^c + (m_{\tilde{D}^c}^2)_{ij} \tilde{D}_i^{\dagger c} \tilde{D}_j^c \\
 & + (m_{\tilde{E}^c}^2)_{ij} \tilde{E}_i^{\dagger c} \tilde{E}_j^c + m_{\tilde{H}_2}^2 \hat{H}_2^\dagger \hat{H}_2 + m_{\tilde{H}_1}^2 \hat{H}_1^\dagger \hat{H}_1 + \left[(h_U)_{ij} \hat{H}_2 \tilde{Q}_i \tilde{U}_j^c \right. \\
 & \quad \left. + (h_D)_{ij} \hat{H}_1 \tilde{Q}_i \tilde{D}_j^c + (h_E)_{ij} \hat{H}_1 \tilde{L}_i \tilde{E}_j^c + h.c. \right] - [B \hat{H}_1 \hat{H}_2 + h.c.] \\
 & + \frac{1}{2} (M_1 \tilde{B} \tilde{B} + M_2 \tilde{W}^3 \tilde{W}^3) + M_2 \tilde{W}^+ \tilde{W}^+ + \frac{1}{2} M_3 \tilde{g}^a \tilde{g}^a \\
 & + \left[\frac{1}{2} \epsilon_{xyz} (h_{U_i})_{jk} \tilde{U}_i^{cx} \tilde{D}_j^{cy} \tilde{D}_k^{cz} + h.c. \right],
 \end{aligned} \tag{2.4}$$

where we have suppressed $SU(2)$ indices. Here \tilde{B}, \tilde{W} and \tilde{g} are the gaugino fields, $\tilde{Q}, \tilde{U}, \tilde{D}$ and \tilde{L}, \tilde{E} are the squark and slepton fields, respectively, and $\hat{H}_{1,2}$ are the $SU(2)$ doublet Higgs fields. Finally, h_E, h_D, h_U , and $(h_{U_i})_{jk}$ are the soft SUSY breaking trilinear couplings.

3. Renormalization group equations and the soft β -functions

Here we review the current state of the β -functions of the MSSM calculation. The gauge β function(s) β_g and the matter multiplet anomalous dimension γ_j^i have been calculated up to four and three loop respectively [12, 13] in the R -parity conserving (RPC) case. The full three loop β -functions including soft β -functions have been presented in [14]. The two loop gauge β functions and anomalous dimensions in the R -parity violating (RPV) case can be seen in [3]. In particular, ref. [2] contained a complete set of one-loop β -functions for RPV parameters.

In this work we present the full two loop beta functions in the baryon number violating (BNV) case. These results could be used to check the effect of two loop β -functions corrections on the sparticle spectrum in the case of BNV scenario.

3.1 β -functions

We now present the results for the Yukawa coupling β -functions and the soft β -functions in the BNV scenario. These results are valid in the DRED' scheme [16] (or indeed the DRED one, which differs from DRED' only when we come to the soft β -functions).

In terms of the anomalous dimensions, the Yukawa β -functions are given by [3]:

$$\frac{d}{dt}(Y_E)_{ij} = (Y_E)_{ik}\gamma_{E_k}^{E_j} + (Y_E)_{ij}\gamma_{H_1}^{H_1} + (Y_E)_{kj}\gamma_{L_k}^{L_i} \quad (3.1)$$

$$\frac{d}{dt}(Y_D)_{ij} = (Y_D)_{ik}\gamma_{D_k}^{D_j} + (Y_D)_{ij}\gamma_{H_1}^{H_1} + (Y_D)_{kj}\gamma_{Q_k}^{Q_i} \quad (3.2)$$

$$\frac{d}{dt}(Y_U)_{ij} = (Y_U)_{ik}\gamma_{U_k}^{U_j} + (Y_U)_{ij}\gamma_{H_2}^{H_2} + (Y_U)_{kj}\gamma_{Q_k}^{Q_i} \quad (3.3)$$

$$\frac{d}{dt}(\Lambda_i)_{jk} = (\Lambda_i)_{jl}\gamma_{D_l}^{D_k} + (\Lambda_i)_{lk}\gamma_{D_l}^{D_j} + (\Lambda_l)_{jk}\gamma_{U_l}^{U_i}. \quad (3.4)$$

At two-loop, the anomalous dimensions are given by:

$$\gamma_{f_j}^{f_i} = \frac{1}{(16\pi^2)}\gamma_{f_j}^{(1)f_i} + \frac{1}{(16\pi^2)^2}\gamma_{f_j}^{(2)f_i} \quad (3.5)$$

Two loop gauge β -functions and anomalous dimensions in the (RPV) case are given in ref. [3]. By using these results and setting $\lambda_{ijk} = \lambda'_{ijk} = 0$ in two loop anomalous dimensions in the RPV case, we have obtained the full-two loop β -functions in the BNV scenario. We have written a Form program which produces the full two loop renormalization group equations for the MSSM in the RPV and BNV scenario, including all soft supersymmetry breaking terms. To test the program we have calculated the full one loop R -parity violating RGEs, and have compared with ref.. [2, 4]. Our results are the same as theirs. Moreover; when we set all R -parity violating couplings to be zero we have obtained the full two loop R -parity conserving RGEs which are consistent with results in ref. [15]. By using the program, we have calculated the RGEs, in the BNV scenario.

The two-loop Yukawa β -functions are:

$$\beta_{(Y_E)_{ij}}^{(2)} = \frac{-6}{(16\pi^2)^2} \{ (Y_E)_{ij} Tr(Y_D^\dagger Y_D \Lambda_{Uq}^\dagger \Lambda_{Uq}) \} + \beta_{(Y_E)(RPC)}^{(2)} \quad (3.6)$$

$$\begin{aligned} \beta_{(Y_D)_{ij}}^{(2)} = \frac{1}{(16\pi^2)^2} \left\{ \left(\frac{16}{3}g_3^2 + \frac{16}{15}g_1^2 \right) (Y_D)_{ik} (\Lambda_{Uq} \Lambda_{Uq}^\dagger)_{jk} \right. \\ - 4(Y_D)_{ik} (\Lambda_{Uq} Y_D^\dagger Y_D \Lambda_{Uq}^\dagger)_{jk} - 4(Y_D)_{ik} (\Lambda_{Uq}^\dagger \Lambda_{Um} \Lambda_{Um}^\dagger \Lambda_{Uq})_{kj} \\ - 2(Y_D)_{ik} (\Lambda_{Um}^\dagger \Lambda_{Ul})_{kj} [Tr(\Lambda_{Ul}^\dagger \Lambda_{Um}) + 2(Y_U^\dagger Y_U)_{lm}] \\ - 6(Y_D)_{ij} Tr(Y_D^\dagger Y_D \Lambda_{Uq}^\dagger \Lambda_{Uq}) - 2(Y_D)_{kj} (Y_D \Lambda_{Uq}^\dagger \Lambda_{Uq} Y_D^\dagger)_{ik} \\ \left. - (Y_D)_{kj} (Y_U)_{il} (Y_U^\dagger)_{mk} Tr(\Lambda_{Ul}^\dagger \Lambda_{Um}) \right\} + \beta_{(Y_D)(RPC)}^{(2)} \quad (3.7) \end{aligned}$$

$$\begin{aligned} \beta_{(Y_U)_{ij}}^{(2)} = \frac{1}{(16\pi^2)^2} \{ (8/3g_3^2 - 4/15g_1^2) (Y_U)_{ik} Tr(\Lambda_{Uk}^\dagger \Lambda_{Uj}) \\ - 4(Y_U)_{ik} Tr(\Lambda_{Uj} \Lambda_{Uk}^\dagger \Lambda_{Uq} \Lambda_{Uq}^\dagger) - 4(Y_U)_{ik} Tr(\Lambda_{Uj} \Lambda_{Uk}^\dagger Y_D^\dagger Y_D) \\ - 3(Y_U)_{ij} (Y_U^\dagger Y_U)_{lm} Tr(\Lambda_{Ul} \Lambda_{Um}^\dagger) - 2(Y_U)_{kj} (Y_D \Lambda_{Uq}^\dagger \Lambda_{Uq} Y_D^\dagger)_{ik} \\ - (Y_U)_{il} (Y_U^\dagger Y_U)_{mj} Tr(\Lambda_{Um} \Lambda_{Ul}^\dagger) \} + \beta_{(Y_U)(RPC)}^{(2)} \quad (3.8) \end{aligned}$$

$$\begin{aligned} \beta_{(\Lambda_{U_i})_{jk}}^{(2)} = \frac{1}{(16\pi^2)^2} \{ (\Lambda_{U_i})_{jk} [64/15g_1^2 g_3^2 + 28/5g_1^4 - 8/3g_3^4] \\ + (6g_2^2 + 2/5g_1^2) (\Lambda_{U_i})_{jl} (Y_D Y_D^\dagger)_{kl} + (16/3g_3^2 + 16/15g_1^2) (\Lambda_{U_i})_{lk} (\Lambda_{Uq} \Lambda_{Uq}^\dagger)_{jl} \end{aligned}$$

$$\begin{aligned}
 & +(16/3g_3^2 + 16/15g_1^2)(\Lambda_{U_i})_{jl}(\Lambda_{U_q}\Lambda_{U_q}^\dagger)_{kl} + (6g_2^2 + 2/5g_1^2)(\Lambda_{U_i})_{lk}(Y_D Y_D^\dagger)_{jl} \\
 & +(6g_2^2 - 2/5g_1^2)(\Lambda_{U_l})_{jk}(Y_U^\dagger Y_U)_{li} + (8/3g_3^2 - 4/15g_1^2)(\Lambda_{U_l})_{jk}Tr(\Lambda_{U_i}\Lambda_{U_l}^\dagger) \\
 & -2(\Lambda_{U_i})_{jl}(Y_D^\dagger Y_D Y_D^\dagger Y_D)_{lk} - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger Y_D Y_D^\dagger Y_D)_{lj} \\
 & -2(\Lambda_{U_i})_{jl}(Y_D^\dagger Y_U Y_U^\dagger Y_D)_{lk} - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger Y_U Y_U^\dagger Y_D)_{lj} \\
 & -2(\Lambda_{U_l})_{jk}(Y_U^\dagger Y_D Y_D^\dagger Y_U)_{li} - 2(\Lambda_{U_l})_{jk}(Y_U^\dagger Y_U Y_U^\dagger Y_U)_{li} \\
 & -2(\Lambda_{U_i})_{jl}(Y_D^\dagger Y_D)_{lk}Tr(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D) - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger Y_D)_{lj}Tr(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D) \\
 & -6(\Lambda_{U_l})_{jk}(Y_U^\dagger Y_U)_{li}Tr(Y_U^\dagger Y_U) - 4(\Lambda_{U_i})_{jl}(\Lambda_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{kl} \\
 & -4(\Lambda_{U_i})_{lk}(\Lambda_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{jl} - 4(\Lambda_{U_l})_{jk}Tr(\Lambda_{U_i} Y_D^\dagger Y_D \Lambda_{U_l}^\dagger) \\
 & -2(\Lambda_{U_i})_{jl}(\Lambda_{U_m}^\dagger \Lambda_{U_n})_{lk}[Tr(\Lambda_{U_n}^\dagger \Lambda_{U_m}) + 2(Y_U^\dagger Y_U)_{nm}] \\
 & -2(\Lambda_{U_i})_{lk}(\Lambda_{U_m}^\dagger \Lambda_{U_n})_{lj}[Tr(\Lambda_{U_n}^\dagger \Lambda_{U_m}) + 2(Y_U^\dagger Y_U)_{nm}] \\
 & -4(\Lambda_{U_i})_{jl}(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{lk} - 4(\Lambda_{U_i})_{lk}(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{lj} \\
 & -4(\Lambda_{U_l})_{jk}Tr(\Lambda_{U_l}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_l}) \} \tag{3.9}
 \end{aligned}$$

The β -function for the Higgs μ -term is given by:

$$\beta_\mu = \mu \{ \gamma_{H_1}^{H_1} + \gamma_{H_2}^{H_2} \} \tag{3.10}$$

so,

$$\beta_\mu^{(2)} = \frac{1}{(16\pi^2)^2} \mu \{ -Tr(6Y_D^\dagger Y_D \Lambda_{U_q}^\dagger \Lambda_{U_q}) - 3(Y_U^\dagger Y_U)_{lk} Tr(\Lambda_{U_k}^\dagger \Lambda_{U_l}) \} + \beta_{\mu(RPC)}^{(2)} \tag{3.11}$$

To complete our results we review the gauge β -functions [2]. They are given by:

$$\beta_{g_1}^{(2)} = \frac{1}{(16\pi^2)^2} \left\{ -\frac{12}{5} g_1^3 Tr(\Lambda_{U_l} \Lambda_{U_l}^\dagger) \right\} + \beta_{g_1(RPC)}^{(2)} \tag{3.12}$$

$$\beta_{g_2}^{(2)} = \beta_{g_2(RPC)}^{(2)} \tag{3.13}$$

$$\beta_{g_3}^{(2)} = \frac{1}{(16\pi^2)^2} \{ -3g_3^3 Tr(\Lambda_{U_l} \Lambda_{U_l}^\dagger) \} + \beta_{g_3(RPC)}^{(2)}. \tag{3.14}$$

By using the method that has been introduced in ref. [17], the two-loop soft β -functions in baryon number violation case can be obtained. They are given by:

$$\begin{aligned}
 \beta_B^{(2)} = \frac{1}{(16\pi^2)^2} \{ & B(-Tr(6Y_D^\dagger Y_D \Lambda_{U_q}^\dagger \Lambda_{U_q}) - 3(Y_U^\dagger Y_U)_{lk} Tr(\Lambda_{U_k}^\dagger \Lambda_{U_l})) \\
 & + 2\mu(-Tr(6Y_D^\dagger h_D \Lambda_{U_q}^\dagger \Lambda_{U_q}) - Tr(6Y_D^\dagger Y_D \Lambda_{U_q}^\dagger h_{U_q})) \\
 & - 3(Y_U^\dagger h_U)_{lk} Tr(\Lambda_{U_k}^\dagger \Lambda_{U_l}) - 3(Y_U^\dagger Y_U)_{lk} Tr(\Lambda_{U_k}^\dagger h_{U_l}) \} + \beta_{B(RPC)}^{(2)} \tag{3.15}
 \end{aligned}$$

$$\beta_{M_1}^{(2)} = \frac{1}{(16\pi^2)^2} \left\{ \frac{12}{5} g_1^2 Tr(h_{U_l} \Lambda_{U_l}^\dagger) - \frac{12}{5} M_1 g_1^2 Tr(\Lambda_{U_l} \Lambda_{U_l}^\dagger) \right\} + \beta_{M_1(RPC)}^{(2)} \tag{3.16}$$

$$\beta_{M_2}^{(2)} = \beta_{M_2(RPC)}^{(2)} \tag{3.17}$$

$$\beta_{M_3}^{(2)} = \frac{1}{(16\pi^2)^2} \{ 3g_3^2 Tr(h_{U_l} \Lambda_{U_l}^\dagger) - 3M_3 g_3^2 Tr(\Lambda_{U_l} \Lambda_{U_l}^\dagger) \} + \beta_{M_3(RPC)}^{(2)} \tag{3.18}$$

$$\beta_{(h_E)ij}^{(2)} = \frac{1}{(16\pi^2)^2} \{-6(h_E)_{ij} \text{Tr}(Y_D^\dagger Y_D \Lambda_{U_q}^\dagger \Lambda_{U_q}) + (Y_E)_{ij} [-12 \text{Tr}(Y_D^\dagger h_D \Lambda_{U_q}^\dagger \Lambda_{U_q}) - 12 \text{Tr}(Y_D^\dagger Y_D \Lambda_{U_q}^\dagger h_{U_q})]\} + \beta_{(h_E)(RPC)}^{(2)} \quad (3.19)$$

$$\begin{aligned} \beta_{(h_D)ij}^{(2)} = & \frac{1}{(16\pi^2)^2} \{(16/3g_3^2 + 16/15g_1^2)(h_D)_{il}(\Lambda_{U_q} \Lambda_{U_q}^\dagger)_{jl} \\ & - 4(h_D)_{il}(\Lambda_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{jl} - 4(h_D)_{il}(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{lj} \\ & - 2(h_D)_{il}(\Lambda_{U_k}^\dagger \Lambda_{U_m})_{lj} [\text{Tr}(\Lambda_{U_m}^\dagger \Lambda_{U_k}) + 2(Y_U^\dagger Y_U)_{mk}] \\ & - 6(h_D)_{ij} \text{Tr}(Y_D^\dagger Y_D \Lambda_{U_q} \Lambda_{U_q}^\dagger) - 2(h_D)_{lj} (Y_D \Lambda_{U_q}^\dagger \Lambda_{U_q} Y_D^\dagger)_{il} \\ & - (h_D)_{lj} (Y_U)_{im} (Y_U^\dagger)_{kl} \text{Tr}(\Lambda_{U_m}^\dagger \Lambda_{U_k}) + 2(16/3M_3^2 g_3^2 \\ & + 16/15M_1^2 g_1^2)(Y_D)_{il}(\Lambda_{U_q} \Lambda_{U_q}^\dagger)_{jl} + 2(16/3g_3^2 + 16/15g_1^2)(Y_D)_{il}(h_{U_q} \Lambda_{U_q}^\dagger)_{jl} \\ & - 8(Y_D)_{il}(h_{U_q} Y_D^\dagger Y_D \Lambda_{U_q}^\dagger)_{jl} - 8(Y_D)_{il}(\Lambda_{U_q} Y_D^\dagger h_D \Lambda_{U_q}^\dagger)_{jl} \\ & - 8(Y_D)_{il}(\Lambda_{U_m}^\dagger h_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{lj} - 8(Y_D)_{il}(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger h_{U_m})_{lj} \\ & - 4(Y_D)_{il}(\Lambda_{U_k}^\dagger h_{U_m})_{lj} [\text{Tr}(\Lambda_{U_m}^\dagger \Lambda_{U_k}) + 2(Y_U^\dagger Y_U)_{mk}] \\ & - 4(Y_D)_{il}(\Lambda_{U_k}^\dagger \Lambda_{U_m})_{lj} [\text{Tr}(\Lambda_{U_m}^\dagger h_{U_k}) + 2(Y_U^\dagger h_U)_{mk}] \\ & - 12(Y_D)_{ij} \text{Tr}(Y_D^\dagger h_D \Lambda_{U_q} \Lambda_{U_q}^\dagger) - 12(Y_D)_{ij} \text{Tr}(Y_D^\dagger Y_D h_{U_q} \Lambda_{U_q}^\dagger) \\ & - 4(Y_D)_{lj} (h_D \Lambda_{U_q}^\dagger \Lambda_{U_q} Y_D^\dagger)_{il} - 4(Y_D)_{lj} (Y_D \Lambda_{U_q}^\dagger h_{U_q} Y_D^\dagger)_{il} \\ & - 2(Y_D)_{lj} (h_U)_{im} (Y_U^\dagger)_{kl} \text{Tr}(\Lambda_{U_m}^\dagger \Lambda_{U_k}) \\ & - 2(Y_D)_{lj} (Y_U)_{im} (Y_U^\dagger)_{kl} \text{Tr}(\Lambda_{U_m}^\dagger h_{U_k})\} + \beta_{(h_D)(RPC)}^{(2)} \end{aligned} \quad (3.20)$$

$$\begin{aligned} \beta_{(h_U)ij}^{(2)} = & \frac{1}{(16\pi^2)^2} \{(h_U)_{il} [(8/3g_3^2 - 4/15g_1^2) \text{Tr}(\Lambda_{U_j} \Lambda_{U_l}^\dagger) \\ & - 4 \text{Tr}(\Lambda_{U_j} \Lambda_{U_l}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger) - 4 \text{Tr}(\Lambda_{U_l}^\dagger \Lambda_{U_j} Y_D^\dagger Y_D)] \\ & - 3(h_U)_{ij} (Y_U^\dagger Y_U)_{lk} \text{Tr}(\Lambda_{U_k}^\dagger \Lambda_{U_l}) - (h_U)_{lj} [2(Y_D \Lambda_{U_q}^\dagger \Lambda_{U_q} Y_D^\dagger)_{il} \\ & + (Y_U)_{im} (Y_U^\dagger)_{kl} \text{Tr}(\Lambda_{U_m}^\dagger \Lambda_{U_k})] \\ & - 2(Y_U)_{il} [(8/3M_3^2 g_3^2 - 4/15M_1^2 g_1^2) \text{Tr}(\Lambda_{U_j} \Lambda_{U_l}^\dagger) \\ & - (8/3g_3^2 - 4/15g_1^2) \text{Tr}(h_{U_j} \Lambda_{U_l}^\dagger) + 4 \text{Tr}(h_{U_j} \Lambda_{U_l}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger) \\ & + 4 \text{Tr}(\Lambda_{U_j} \Lambda_{U_l}^\dagger h_{U_q} \Lambda_{U_q}^\dagger) + 4 \text{Tr}(\Lambda_{U_l}^\dagger h_{U_j} Y_D^\dagger Y_D) \\ & + 4 \text{Tr}(\Lambda_{U_l}^\dagger \Lambda_{U_j} Y_D^\dagger h_D)] - 2(Y_U)_{ij} [3(Y_U^\dagger h_U)_{lk} \text{Tr}(\Lambda_{U_k}^\dagger \Lambda_{U_l}) \\ & + 3(Y_U^\dagger Y_U)_{lk} \text{Tr}(\Lambda_{U_k}^\dagger h_{U_l})] - 2(Y_U)_{lj} [2(Y_D \Lambda_{U_q}^\dagger h_{U_q} Y_D^\dagger)_{il} \\ & + 2(h_D \Lambda_{U_q}^\dagger \Lambda_{U_q} Y_D^\dagger)_{il} + (h_U)_{im} (Y_U^\dagger)_{kl} \text{Tr}(\Lambda_{U_m}^\dagger \Lambda_{U_k}) \\ & + (Y_U)_{im} (Y_U^\dagger)_{kl} \text{Tr}(\Lambda_{U_m}^\dagger h_{U_k})]\} + \beta_{(h_U)(RPC)}^{(2)} \end{aligned} \quad (3.21)$$

$$\begin{aligned} \beta_{(h_{U_i})jk}^{(2)} = & \frac{1}{(16\pi^2)^2} \{(h_{U_i})_{jk} [64/15g_1^2 g_3^2 + 28/5g_1^4 - 8/3g_3^4] \\ & + (6g_2^2 + 2/5g_1^2)(h_{U_i})_{jl} (Y_D Y_D^\dagger)_{kl} + (16/3g_3^2 + 16/15g_1^2)(h_{U_i})_{lk} (\Lambda_{U_q} \Lambda_{U_q}^\dagger)_{jl} \\ & + (16/3g_3^2 + 16/15g_1^2)(h_{U_i})_{jl} (\Lambda_{U_q} \Lambda_{U_q}^\dagger)_{kl} + (6g_2^2 + 2/5g_1^2)(h_{U_i})_{lk} (Y_D Y_D^\dagger)_{jl} \\ & + (6g_2^2 - 2/5g_1^2)(h_{U_l})_{jk} (Y_U^\dagger Y_U)_{li} + (8/3g_3^2 - 4/15g_1^2)(h_{U_l})_{jk} \text{Tr}(\Lambda_{U_i} \Lambda_{U_l}^\dagger) \end{aligned}$$

$$\begin{aligned}
& -2(h_{U_i})_{jl}(Y_D^\dagger Y_D Y_D^\dagger Y_D)_{lk} - 2(h_{U_i})_{lk}(Y_D^\dagger Y_D Y_D^\dagger Y_D)_{lj} \\
& -2(h_{U_i})_{jl}(Y_D^\dagger Y_U Y_U^\dagger Y_D)_{lk} \\
& -2(h_{U_i})_{lk}(Y_D^\dagger Y_U Y_U^\dagger Y_D)_{lj} - 2(h_{U_i})_{jk}(Y_U^\dagger Y_D Y_D^\dagger Y_U)_{li} \\
& -2(h_{U_i})_{jk}(Y_U^\dagger Y_U Y_U^\dagger Y_U)_{li} - 2(h_{U_i})_{jl}(Y_D^\dagger Y_D)_{lk} Tr(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D) \\
& -2(h_{U_i})_{lk}(Y_D^\dagger Y_D)_{lj} Tr(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D) - 6(h_{U_i})_{jk}(Y_U^\dagger Y_U)_{li} Tr(Y_U^\dagger Y_U) \\
& -4(h_{U_i})_{jl}(\Lambda_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{kl} - 4(h_{U_i})_{lk}(\Lambda_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{jl} \\
& -4(h_{U_i})_{jk} Tr(\Lambda_{U_i} Y_D^\dagger Y_D \Lambda_{U_l}^\dagger) \\
& -2(h_{U_i})_{jl}(\Lambda_{U_m}^\dagger \Lambda_{U_n})_{lk} [Tr(\Lambda_{U_n}^\dagger \Lambda_{U_m}) + 2(Y_U^\dagger Y_U)_{nm}] \\
& -2(h_{U_i})_{lk}(\Lambda_{U_m}^\dagger \Lambda_{U_n})_{lj} [Tr(\Lambda_{U_n}^\dagger \Lambda_{U_m}) + 2(Y_U^\dagger Y_U)_{nm}] \\
& -4(h_{U_i})_{jl}(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{lk} \\
& -4(h_{U_i})_{lk}(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{lj} - 4(h_{U_i})_{jk} Tr(\Lambda_{U_l}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_i}) \\
& + 2[-(\Lambda_{U_i})_{jk}(64/15(M_1^2 + M_2^2)g_1^2 g_3^2 + 56/5M_1^2 g_1^4 - 16/3M_3^2 g_3^4) \\
& - (6M_2^2 g_2^2 + 2/5M_1^2 g_1^2)(\Lambda_{U_i})_{jl}(Y_D Y_D^\dagger)_{kl} + (6g_2^2 + 2/5g_1^2)(\Lambda_{U_i})_{jl}(h_D Y_D^\dagger)_{kl} \\
& - (16/3M_3^2 g_3^2 + 16/15M_1^2 g_1^2)(\Lambda_{U_i})_{lk}(\Lambda_{U_q} \Lambda_{U_q}^\dagger)_{jl} \\
& + (16/3g_3^2 + 16/15g_1^2)(\Lambda_{U_i})_{lk}(h_{U_q} \Lambda_{U_q}^\dagger)_{jl} \\
& - (16/3M_3^2 g_3^2 + 16/15M_1^2 g_1^2)(\Lambda_{U_i})_{jl}(\Lambda_{U_q} \Lambda_{U_q}^\dagger)_{kl} \\
& + (16/3g_3^2 + 16/15g_1^2)(\Lambda_{U_i})_{jl}(h_{U_q} \Lambda_{U_q}^\dagger)_{kl} \\
& - (6M_2^2 g_2^2 + 2/5M_1^2 g_1^2)(\Lambda_{U_i})_{lk}(Y_D Y_D^\dagger)_{jl} + (6g_2^2 + 2/5g_1^2)(\Lambda_{U_i})_{lk}(h_D Y_D^\dagger)_{jl} \\
& - (6M_2^2 g_2^2 - 2/5M_1^2 g_1^2)(\Lambda_{U_l})_{jk}(Y_U^\dagger Y_U)_{li} + (6g_2^2 - 2/5g_1^2)(\Lambda_{U_l})_{jk}(Y_U^\dagger h_U)_{li} \\
& - (8/3M_2^2 g_3^2 - 4/15M_1^2 g_1^2)(\Lambda_{U_l})_{jk} Tr(\Lambda_{U_i} \Lambda_{U_l}^\dagger) \\
& + (8/3g_3^2 - 4/15g_1^2)(\Lambda_{U_l})_{jk} Tr(h_{U_i} \Lambda_{U_l}^\dagger) \\
& - 2(\Lambda_{U_i})_{jl}(Y_D^\dagger h_D Y_D^\dagger Y_D)_{lk} - 2(\Lambda_{U_i})_{jl}(Y_D^\dagger Y_D Y_D^\dagger h_D)_{lk} \\
& - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger h_D Y_D^\dagger Y_D)_{lj} \\
& - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger Y_D Y_D^\dagger h_D)_{lj} - 2(\Lambda_{U_i})_{jl}(Y_D^\dagger h_U Y_U^\dagger Y_D)_{lk} \\
& - 2(\Lambda_{U_i})_{jl}(Y_D^\dagger Y_U Y_U^\dagger h_D)_{lk} - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger h_U Y_U^\dagger Y_D)_{lj} \\
& - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger Y_U Y_U^\dagger h_D)_{lj} - 2(\Lambda_{U_l})_{jk}(Y_U^\dagger h_D Y_D^\dagger Y_U)_{li} \\
& - 2(\Lambda_{U_l})_{jk}(Y_U^\dagger Y_D Y_D^\dagger h_U)_{li} - 2(\Lambda_{U_l})_{jk}(Y_U^\dagger h_U Y_U^\dagger Y_U)_{li} - 2(\Lambda_{U_l})_{jk}(Y_U^\dagger Y_U Y_U^\dagger h_U)_{li} \\
& - 2(\Lambda_{U_i})_{jl}(Y_D^\dagger h_D)_{lk} Tr(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D) - 2(\Lambda_{U_i})_{jl}(Y_D^\dagger Y_D)_{lk} Tr(Y_E^\dagger h_E + 3Y_D^\dagger h_D) \\
& - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger h_D)_{lj} Tr(Y_E^\dagger Y_E + 3Y_D^\dagger Y_D) - 2(\Lambda_{U_i})_{lk}(Y_D^\dagger Y_D)_{lj} Tr(Y_E^\dagger h_E + 3Y_D^\dagger h_D) \\
& - 6(\Lambda_{U_l})_{jk}(Y_U^\dagger h_U)_{li} Tr(Y_U^\dagger Y_U) - 6(\Lambda_{U_l})_{jk}(Y_U^\dagger Y_U)_{li} Tr(Y_U^\dagger h_U) \\
& - 4(\Lambda_{U_i})_{jl}(h_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{kl} \\
& - 4(\Lambda_{U_i})_{jl}(\Lambda_{U_m} Y_D^\dagger h_D \Lambda_{U_m}^\dagger)_{kl} - 4(\Lambda_{U_i})_{lk}(h_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{jl} \\
& - 4(\Lambda_{U_i})_{lk}(\Lambda_{U_m} Y_D^\dagger h_D \Lambda_{U_m}^\dagger)_{jl} \\
& - 4(\Lambda_{U_l})_{jk} Tr(h_{U_i} Y_D^\dagger Y_D \Lambda_{U_l}^\dagger) - 4(\Lambda_{U_l})_{jk} Tr(\Lambda_{U_i} Y_D^\dagger h_D \Lambda_{U_l}^\dagger) \\
& - 2(\Lambda_{U_i})_{jl}(\Lambda_{U_m}^\dagger h_{U_n})_{lk} (Tr(\Lambda_{U_n}^\dagger \Lambda_{U_m}) + 2(Y_U^\dagger Y_U)_{nm})
\end{aligned}$$

$$\begin{aligned}
 & -2(\Lambda_{U_i})_{jl}(\Lambda_{U_m}^\dagger \Lambda_{U_n})_{lk} (Tr(\Lambda_{U_n}^\dagger h_{U_m}) + 2(Y_U^\dagger h_U)_{nm}) \\
 & -2(\Lambda_{U_i})_{lk}(\Lambda_{U_m}^\dagger h_{U_n})_{lj} (Tr(\Lambda_{U_n}^\dagger \Lambda_{U_m}) + 2(Y_U^\dagger Y_U)_{nm}) \\
 & -2(\Lambda_{U_i})_{lk}(\Lambda_{U_m}^\dagger \Lambda_{U_n})_{lj} (Tr(\Lambda_{U_n}^\dagger h_{U_m}) + 2(Y_U^\dagger h_U)_{nm}) \\
 & -4(\Lambda_{U_i})_{jl}(\Lambda_{U_m}^\dagger h_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{lk} - 4(\Lambda_{U_i})_{jl}(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger h_{U_m})_{lk} \\
 & -4(\Lambda_{U_i})_{lk}(\Lambda_{U_m}^\dagger h_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{lj} - 4(\Lambda_{U_i})_{lk}(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger h_{U_m})_{lj} \\
 & -4(\Lambda_{U_l})_{jk} Tr(\Lambda_{U_l}^\dagger h_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_i}) - 4(\Lambda_{U_l})_{jk} Tr(\Lambda_{U_l}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger h_{U_i}) \} \quad (3.22)
 \end{aligned}$$

The RGEs for the soft breaking masses are given by:

$$\begin{aligned}
 \beta_{(M_U^2)ij}^{(2)} = & \frac{1}{(16\pi^2)^2} \left\{ \left[-\frac{16}{15} M_1^2 g_1^2 + \frac{32}{3} M_3^2 g_3^2 \right] Tr(\Lambda_{U_j} \Lambda_{U_i}^\dagger) \right. \\
 & + \left[-\frac{4}{15} g_1^2 + \frac{8}{3} g_3^2 \right] Tr(\Lambda_{U_j} \Lambda_{U_i}^\dagger) (M_{\tilde{U}}^2)_{il} + \left[-\frac{8}{15} g_1^2 + \frac{16}{3} g_3^2 \right] (\Lambda_{U_j} \Lambda_{U_i}^\dagger)_{kl} (M_D^2)_{kl} \\
 & + \left[-\frac{8}{15} g_1^2 + \frac{16}{3} g_3^2 \right] Tr(\Lambda_{U_j} \Lambda_{U_i}^\dagger M_D^2) + \left[-\frac{4}{15} g_1^2 + \frac{8}{3} g_3^2 \right] Tr(\Lambda_{U_l} \Lambda_{U_i}^\dagger) (M_{\tilde{U}}^2)_{lj} \\
 & -4Tr(\Lambda_{U_i} \Lambda_{U_k}^\dagger \Lambda_{U_k} \Lambda_{U_l}^\dagger) (M_{\tilde{U}}^2)_{jl} - 4Tr(\Lambda_{U_l} \Lambda_{U_k}^\dagger \Lambda_{U_k} \Lambda_{U_j}^\dagger) (M_{\tilde{U}}^2)_{li} \\
 & -8Tr(\Lambda_{U_i} \Lambda_{U_l}^\dagger \Lambda_{U_k} \Lambda_{U_j}^\dagger) (M_{\tilde{U}}^2)_{kl} - 8(\Lambda_{U_i} \Lambda_{U_k}^\dagger \Lambda_{U_k} \Lambda_{U_j}^\dagger)_{ql} (M_D^2)_{ql} \\
 & -16Tr(\Lambda_{U_i} \Lambda_{U_k}^\dagger M_D^2 \Lambda_{U_k} \Lambda_{U_j}^\dagger) \\
 & -8(\Lambda_{U_k} \Lambda_{U_j}^\dagger \Lambda_{U_i} \Lambda_{U_k}^\dagger)_{lq} (M_D^2)_{lq} - 8(Y_D \Lambda_{U_i}^\dagger \Lambda_{U_j} Y_D^\dagger)_{kl} (M_Q^2)_{kl} \\
 & -16Tr(Y_D \Lambda_{U_i}^\dagger M_D^2 \Lambda_{U_j} Y_D^\dagger) - 4Tr(Y_D \Lambda_{U_l}^\dagger \Lambda_{U_j} Y_D^\dagger) (M_{\tilde{U}}^2)_{il} \\
 & -4Tr(Y_D \Lambda_{U_i}^\dagger \Lambda_{U_l} Y_D^\dagger) (M_{\tilde{U}}^2)_{lj} \\
 & -8M_{H_1}^2 Tr(Y_D \Lambda_{U_i}^\dagger \Lambda_{U_j} Y_D^\dagger) - 8(\Lambda_{U_j} Y_D^\dagger Y_D \Lambda_{U_i}^\dagger)_{kl} (M_{\tilde{U}}^2)_{kl} \\
 & + \left[\frac{8}{15} M_1 g_1^2 - \frac{16}{3} M_3 g_3^2 \right] Tr(\Lambda_{U_j} h_{U_i}^\dagger) \\
 & + \left[\frac{8}{15} M_1 g_1^2 - \frac{16}{3} M_3 g_3^2 \right] Tr(h_{U_j} \Lambda_{U_i}^\dagger) - 8Tr(\Lambda_{U_i} h_{U_l}^\dagger h_{U_l} \Lambda_{U_j}^\dagger) - 8Tr(\Lambda_{U_i} \Lambda_{U_l}^\dagger h_{U_l} h_{U_j}^\dagger) \\
 & -8Tr(h_{U_i} \Lambda_{U_l}^\dagger \Lambda_{U_l} h_{U_j}^\dagger) - 8Tr(h_{U_i} h_{U_l}^\dagger \Lambda_{U_l} \Lambda_{U_j}^\dagger) \\
 & -8Tr(h_D h_{U_i}^\dagger \Lambda_{U_j} Y_D^\dagger) \\
 & -8Tr(h_D \Lambda_{U_i}^\dagger \Lambda_{U_j} h_D^\dagger) - 8Tr(Y_D \Lambda_{U_i}^\dagger h_{U_j} h_D^\dagger) - 8Tr(Y_D h_{U_i}^\dagger h_{U_j} Y_D^\dagger) \\
 & + \left[-\frac{8}{15} g_1^2 + \frac{16}{3} g_3^2 \right] Tr(h_{U_j} h_{U_i}^\dagger) \} + \beta_{(M_{\tilde{U}}^2)RPC}^{(2)} \quad (3.23)
 \end{aligned}$$

$$\begin{aligned}
 \beta_{(M_Q^2)ij}^{(2)} = & \frac{1}{(16\pi^2)^2} \{ -2Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) (M_{\tilde{U}}^2)_{km} (Y_U)_{jl} (Y_U^\dagger)_{mi} - 2Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) (M_{\tilde{U}}^2)_{ml} (Y_U)_{jm} (Y_U^\dagger)_{ki} \\
 & -4(\Lambda_{U_k}^\dagger \Lambda_{U_q})_{ml} (M_{\tilde{U}}^2)_{qk} (Y_b)_{jm} (Y_b^\dagger)_{li} - 2(\Lambda_{U_l}^\dagger \Lambda_{U_k})_{mn} (M_D^2)_{nm} (Y_U)_{jl} (Y_U^\dagger)_{ki} \\
 & -2(\Lambda_{U_k} \Lambda_{U_l}^\dagger)_{mn} (M_D^2)_{mn} (Y_U)_{jl} (Y_U^\dagger)_{ki} - 4(\Lambda_{U_q}^\dagger \Lambda_{U_q})_{ml} (M_D^2)_{lk} (Y_D)_{jm} (Y_D^\dagger)_{ki} \\
 & -4(\Lambda_{U_q}^\dagger \Lambda_{U_q})_{ml} (M_D^2)_{km} (Y_D)_{jk} (Y_D^\dagger)_{li} - 4(\Lambda_{U_q} Y_D^\dagger)_{ni} (M_D^2)_{nk} (Y_D \Lambda_{U_q}^\dagger)_{jk} \\
 & -Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) (M_Q^2)_{im} (Y_U)_{jl} (Y_U^\dagger)_{km} - Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) (M_Q^2)_{mj} (Y_U)_{ml} (Y_U^\dagger)_{ki}
 \end{aligned}$$

$$\begin{aligned}
 & -2(\Lambda_{U_q}^\dagger \Lambda_{U_q})_{ml} (M_{\tilde{Q}}^2)_{ik} (Y_D)_{jm} (Y_D^\dagger)_{lk} - 2(\Lambda_{U_q}^\dagger \Lambda_{U_q})_{ml} (M_{\tilde{Q}}^2)_{kj} (Y_D)_{km} (Y_D^\dagger)_{li} \\
 & -2Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) [(h_U)_{jl} (h_{U_l}^\dagger)_{ki} + M_{H_2}^2 (Y_U)_{jl} (Y_U^\dagger)_{ki}] - 2Tr(\Lambda_{U_k} h_{U_l}^\dagger) (h_U)_{jl} (Y_U^\dagger)_{ki} \\
 & -4(\Lambda_{U_q} \Lambda_{U_q}^\dagger)_{ml} [(h_D)_{jm} (h_D^\dagger)_{li} + M_{H_1}^2 (Y_D)_{jm} (Y_D^\dagger)_{li}] - 4(\Lambda_{U_q} Y_D^\dagger)_{ni} (h_D h_{U_q}^\dagger)_{jn} \\
 & -4(\Lambda_{U_q}^\dagger h_{U_q})_{ml} (Y_D)_{jm} (h_D^\dagger)_{li} - 2Tr(\Lambda_{U_l}^\dagger h_{U_k}) (Y_U)_{jl} (h_{U_l}^\dagger)_{ki} \\
 & -2Tr(h_{U_l}^\dagger h_{U_k}) (Y_U)_{jl} (Y_U^\dagger)_{ki} - 4(h_{U_q}^\dagger h_{U_q})_{ml} (Y_D)_{jm} (Y_D^\dagger)_{li} \} + \beta_{(M_{\tilde{Q}}^2)RPC}^{(2)} \quad (3.24)
 \end{aligned}$$

$$\begin{aligned}
 \beta_{(M_{H_1}^2)}^{(2)} &= \frac{1}{(16\pi^2)^2} \{ = -12(Y_D)_{ln} (\Lambda_{U_q}^\dagger \Lambda_{U_q} Y_D^\dagger)_{nm} (M_{\tilde{Q}}^2)_{lm} - 24Tr(\Lambda_{U_q}^\dagger \Lambda_{U_q} M_D^2 Y_D^\dagger Y_D) \\
 & -12Tr(\Lambda_{U_l}^\dagger \Lambda_{U_q} Y_D^\dagger Y_D) (M_{\tilde{U}}^2)_{ql} - 12(\Lambda_{U_q}^\dagger)_{nl} (M_D^2)_{kl} (\Lambda_{U_q} Y_D^\dagger Y_D)_{kn} \\
 & -12M_{H_1}^2 Tr(\Lambda_{U_q}^\dagger \Lambda_{U_q} Y_D^\dagger Y_D) - 12Tr(\Lambda_{U_q}^\dagger \Lambda_{U_q} h_D^\dagger h_D) \\
 & -12Tr(h_{U_q}^\dagger \Lambda_{U_q} Y_D^\dagger h_D) - 12Tr(\Lambda_{U_q}^\dagger h_{U_q} h_D^\dagger Y_D) - 12Tr(h_{U_q}^\dagger h_{U_q} Y_D^\dagger Y_D) \} + \beta_{(M_{H_1}^2)RPC}^{(2)} \quad (3.25)
 \end{aligned}$$

$$\begin{aligned}
 \beta_{(M_{H_2}^2)}^{(2)} &= \frac{1}{(16\pi^2)^2} \{ -3Tr(\Lambda_{U_k}^\dagger \Lambda_{U_l}) (Y_U Y_U^\dagger)_{ml} (M_{\tilde{Q}}^2)_{km} - 3Tr(\Lambda_{U_k}^\dagger \Lambda_{U_l}) (Y_U Y_U^\dagger)_{mk} (M_{\tilde{Q}}^2)_{ml} \\
 & -6Tr(\Lambda_{U_k}^\dagger \Lambda_{U_l}) (Y_U M_{\tilde{U}}^2 Y_U^\dagger)_{lk} - 6M_{H_2}^2 Tr(\Lambda_{U_k}^\dagger \Lambda_{U_l}) (Y_U Y_U^\dagger)_{lk} \\
 & -3Tr(\Lambda_{U_m}^\dagger \Lambda_{U_l}) (Y_U Y_U^\dagger)_{lk} (M_{\tilde{U}}^2)_{km} - 3Tr(\Lambda_{U_k}^\dagger \Lambda_{U_m}) (Y_U Y_U^\dagger)_{lk} (M_{\tilde{U}}^2)_{ml} \\
 & -6Tr(\Lambda_{U_k}^\dagger M_D^2 \Lambda_{U_l}) (Y_U Y_U^\dagger)_{lk} - 6(M_D^2)_{mq} (\Lambda_{U_l} \Lambda_{U_k}^\dagger)_{mq} (Y_U Y_U^\dagger)_{lk} \\
 & -6Tr(\Lambda_{U_k}^\dagger \Lambda_{U_l}) (h_U h_{U_l}^\dagger)_{lk} - 6Tr(h_{U_k}^\dagger \Lambda_{U_l}) (h_U Y_U^\dagger)_{lk} \\
 & -6Tr(\Lambda_{U_k}^\dagger h_{U_l}) (Y_U h_{U_l}^\dagger)_{lk} - 6Tr(h_{U_k}^\dagger h_{U_l}) (Y_U Y_U^\dagger)_{lk} \} + \beta_{(M_{H_2}^2)RPC}^{(2)} \quad (3.26)
 \end{aligned}$$

$$\begin{aligned}
 \beta_{(M_D^2)ij}^{(2)} &= \frac{1}{(16\pi^2)^2} \left\{ \left(\frac{16}{15} g_1^2 + \frac{16}{3} g_3^2 \right) (\Lambda_{U_m} \Lambda_{U_m}^\dagger)_{jq} (M_D^2)_{iq} + \left(\frac{16}{15} g_1^2 + \frac{16}{3} g_3^2 \right) (\Lambda_{U_m} \Lambda_{U_m}^\dagger)_{qi} (M_D^2)_{qj} \right. \\
 & + \left(\frac{32}{15} g_1^2 + \frac{32}{3} g_3^2 \right) (\Lambda_{U_m} M_D^2 \Lambda_{U_m}^\dagger)_{ji} + \left(\frac{32}{15} g_1^2 + \frac{32}{3} g_3^2 \right) (\Lambda_{U_m} \Lambda_{U_q}^\dagger)_{ji} (M_{\tilde{U}}^2)_{mq} \\
 & + \left(\frac{64}{15} M_1^2 g_1^2 + \frac{64}{3} M_3^2 g_3^2 \right) (\Lambda_{U_q} \Lambda_{U_q}^\dagger)_{ji} - 8(\Lambda_{U_m} M_D^2 Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{ji} \\
 & - 8(\Lambda_{U_m} Y_D^\dagger Y_D M_D^2 \Lambda_{U_m}^\dagger)_{ji} - 4(\Lambda_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{jq} (M_D^2)_{iq} \\
 & - 4(\Lambda_{U_m} Y_D^\dagger Y_D)_{qk} (\Lambda_{U_m}^\dagger)_{ki} (M_D^2)_{qj} - 8(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger)_{ik} (\Lambda_{U_m})_{lj} (M_D^2)_{lk} \\
 & - 8(\Lambda_{U_m}^\dagger \Lambda_{U_q} M_D^2 \Lambda_{U_q}^\dagger \Lambda_{U_m})_{ij} - 8(\Lambda_{U_m}^\dagger)_{ik} (M_D^2)_{nk} (\Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{nj} \\
 & - 4(M_D^2 \Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{ij} - 4(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m} M_D^2)_{ij} \\
 & - 4(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} (\Lambda_{U_k} \Lambda_{U_l}^\dagger)_{qm} (M_D^2)_{qm} - 4(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger M_D^2) \\
 & - 4(\Lambda_{U_k}^\dagger)_{iq} (M_D^2)_{mq} (\Lambda_{U_l})_{mj} Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) - 2(M_D^2 \Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) \\
 & - 2(\Lambda_{U_k}^\dagger \Lambda_{U_l} M_D^2)_{ij} Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) - 8(\Lambda_{U_k}^\dagger)_{iq} (M_D^2)_{mq} (\Lambda_{U_l})_{mj} (Y_U^\dagger Y_U)_{lk} \\
 & - 4(M_D^2 \Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} (Y_U^\dagger Y_U)_{lk} - 4(\Lambda_{U_k}^\dagger \Lambda_{U_l} M_D^2)_{ij} (Y_U^\dagger Y_U)_{lk} \\
 & - 8(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} (Y_U^\dagger)_{lq} (Y_U)_{nk} (M_{\tilde{Q}}^2)_{nq} - 8(\Lambda_{U_m} Y_D^\dagger)_{jq} (Y_D \Lambda_{U_m}^\dagger)_{li} (M_{\tilde{Q}}^2)_{lq} \\
 & - 8M_{H_1}^2 (\Lambda_{U_m} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{ji} - 8(\Lambda_{U_q} Y_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{ji} (M_{\tilde{U}}^2)_{qm} \\
 & \left. - 8(\Lambda_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{ij} (M_{\tilde{U}}^2)_{qk} - 8(\Lambda_{U_k}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{ij} (M_{\tilde{U}}^2)_{mk} \right\}
 \end{aligned}$$

$$\begin{aligned}
 & -2(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} Tr(\Lambda_{U_k} \Lambda_{U_q}^\dagger)(M_{\tilde{U}}^2)_{lq} - 2(\Lambda_{U_q}^\dagger \Lambda_{U_l})_{ij} Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger)(M_{\tilde{U}}^2)_{kq} \\
 & -2(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} Tr(\Lambda_{U_q} \Lambda_{U_l}^\dagger)(M_{\tilde{U}}^2)_{qk} - 2(\Lambda_{U_k}^\dagger \Lambda_{U_q})_{ij} Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger)(M_{\tilde{U}}^2)_{ql} \\
 & -4(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} (Y_U^\dagger Y_U)_{qk} (M_{\tilde{U}}^2)_{lq} - 4(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} (Y_U^\dagger Y_U)_{lq} (M_{\tilde{U}}^2)_{qk} \\
 & -8M_{H_2}^2 (\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} (Y_U^\dagger Y_U)_{lk} - 4(\Lambda_{U_q}^\dagger \Lambda_{U_l})_{ij} (Y_U^\dagger Y_U)_{lk} (M_{\tilde{U}}^2)_{kq} \\
 & -4(\Lambda_{U_k}^\dagger \Lambda_{U_q})_{ij} (Y_U^\dagger Y_U)_{lk} (M_{\tilde{U}}^2)_{ql} - \left(\frac{32}{15} M_1 g_1^2 + \frac{32}{3} M_3 g_3^2 \right) (\Lambda_{U_m} h_{U_m}^\dagger)_{ji} \\
 & - \left(\frac{32}{15} M_1 g_1^2 + \frac{32}{3} M_3 g_3^2 \right) (h_{U_m} \Lambda_{U_m}^\dagger)_{ji} \\
 & + \left(\frac{32}{15} g_1^2 + \frac{32}{3} g_3^2 \right) (h_{U_m} h_{U_m}^\dagger)_{ji} - 8(\Lambda_{U_m}^\dagger h_{U_q} h_{U_q}^\dagger \Lambda_{U_m})_{ij} \\
 & - 8(h_{U_m}^\dagger h_{U_q} \Lambda_{U_q}^\dagger \Lambda_{U_m})_{ij} - 8(\Lambda_{U_m}^\dagger \Lambda_{U_q} h_{U_q}^\dagger h_{U_m})_{ij} \\
 & - 8(h_{U_m}^\dagger \Lambda_{U_q} \Lambda_{U_q}^\dagger h_{U_m})_{ij} - 8(\Lambda_{U_m} h_D^\dagger h_D \Lambda_{U_m}^\dagger)_{ji} - 8(\Lambda_{U_m} Y_D^\dagger h_D h_{U_m}^\dagger)_{ji} \\
 & - 8(h_{U_m} h_D^\dagger Y_D \Lambda_{U_m}^\dagger)_{ji} - 8(h_{U_m} Y_D^\dagger Y_D h_{U_m}^\dagger)_{ji} \\
 & - 8(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} (h_U^\dagger h_U)_{lk} - 8(h_{U_k}^\dagger h_{U_l})_{ij} (Y_U^\dagger Y_U)_{lk} \\
 & - 8(h_{U_k}^\dagger \Lambda_{U_l})_{ij} (Y_U^\dagger h_U)_{lk} - 8(\Lambda_{U_k}^\dagger h_{U_l})_{ij} (h_U^\dagger Y_U)_{lk} \\
 & - 4(\Lambda_{U_k}^\dagger \Lambda_{U_l})_{ij} Tr(h_{U_k} h_{U_l}^\dagger) - 4(h_{U_k}^\dagger h_{U_l})_{ij} Tr(\Lambda_{U_k} \Lambda_{U_l}^\dagger) \\
 & - 4(h_{U_k}^\dagger \Lambda_{U_l})_{ij} Tr(h_{U_k} \Lambda_{U_l}^\dagger) - 4(\Lambda_{U_k}^\dagger h_{U_l})_{ij} Tr(h_{U_k} \Lambda_{U_l}^\dagger) \Big\} + \beta_{(M_D^2)RPC}^{(2)} \quad (3.27)
 \end{aligned}$$

$$\beta_{(M_E^2)ij}^{(2)} = \beta_{(M_E^2)RPC}^{(2)} \quad (3.28)$$

$$\beta_{(M_L^2)ij}^{(2)} = \beta_{(M_L^2)RPC}^{(2)} \quad (3.29)$$

4. The numerical calculation of the sparticle masses

Apart from λ''_{211} and λ''_{311} which have been strongly bounded [18], the bounds on the λ'' are much weak, and in general come from the requirement of a perturbative behavior. It is worthwhile to investigate the effects of using two-loop than one-loop β -functions in the BNV scenario. To simplify the numerical calculation we have made several simplifications. we ignore all λ'' terms in the Lagrangian except one (λ''_{233} in one case and λ''_{212} in another case). We have also three regular Yukawa terms which come from the third generation and ignore their lower generation counterparts.

In order to examine the effect of the two loop corrections of the β -functions in the BNV scenario on the sparticle masses, we solve the renormalization group equations. We have focused on the standard treatment with universal boundary conditions at gauge unification, often named mSUGRA. Thus we assume that at M_X we have universal soft scalar masses (m_0), gaugino masses ($m_{1/2}$) and A -parameters (A). We calculate the appropriate dimensionless coupling input values at M_Z in the dimensional reduction regularization scheme (\overline{DR}) from the Standard Model \overline{MS} gauge couplings and the physical quark masses by incorporating supersymmetric threshold corrections in the manner of ref. [19].

mass	1-loop $\lambda''_{ijk} = 0$	2-loop $\lambda''_{ijk} = 0$	1-loop $\lambda''_{323} = .87$	2-loop $\lambda''_{323} = .87$	1-loop $\lambda''_{212} = .5$	2-loop $\lambda''_{212} = .5$
\tilde{g}	628	613	624	614	625	613
\tilde{t}_1	400	399	306	306	400	399
\tilde{t}_2	595	591	574	572	595	591
\tilde{u}_L	573	565	573	565	573	565
\tilde{u}_R	552	548	552	548	552	548
\tilde{c}_L	573	565	574	568	574	565
\tilde{c}_R	552	548	553	551	504	501
\tilde{b}_1	521	515	435	433	521	515
\tilde{b}_2	551	548	532	530	551	548
\tilde{d}_L	579	571	579	571	579	571
\tilde{d}_R	551	548	551	548	506	503
\tilde{s}_L	579	571	580	574	579	571
\tilde{s}_R	551	548	440	438	506	503
$\tilde{\tau}_1$	212	207	212	206	212	207
$\tilde{\tau}_2$	139	135	138	135	139	135
\tilde{e}_L	209	203	209	203	209	203
\tilde{e}_R	147	144	146	143	147	144
$\tilde{\nu}_e$	192	186	193	186	192	186
$\tilde{\nu}_\tau$	191	185	192	185	191	185
χ_1	104	97	103	97	104	97
χ_2	193	179	188	176	193	179
χ_3	345	362	317	332	345	362
χ_4	370	382	347	356	370	382
χ_1^\pm	192	178	187	175	192	178
χ_2^\pm	370	383	346	356	370	383
h	112	112	110	110	112	112
H	386	397	362	371	386	397
A	386	397	362	371	386	397
H^\pm	396	406	371	380	396	406

Table 1: Sparticle masses (in GeV) for the SPS1a point with $m_t = 174.3$ GeV in the R -parity conserving case (column two), and in the BNV scenario (column three and four). Supersymmetric masses have been calculated for first and second loop corrections in each scenario.

We run all dimensionless couplings from M_Z to M_X (the unification scale, defined as where α_1 and α_2 meet), then impose the boundary conditions on the soft parameters and masses. Finally, we run down all the couplings from M_X to M_Z , and compute the sparticle spectrum (for the calculations stop masses we incorporate one-loop threshold corrections, and evaluate the sparticle masses at their own scale.). Because of the interdependence of the boundary condition at the M_Z and M_X we determine the couplings by an iterative

mass	1-loop $\lambda''_{ijk} = 0$	2-loop $\lambda''_{ijk} = 0$	1-loop $\lambda''_{323} = .87$	2-loop $\lambda''_{323} = .87$
\tilde{g}	628	613	624	614
\tilde{t}_1	401	399	305	305
\tilde{t}_2	596	592	575	573
\tilde{u}_L	573	565	573	565
\tilde{u}_R	552	548	552	548
\tilde{c}_L	573	565	574	568
\tilde{c}_R	552	548	553	551
\tilde{b}_1	522	516	435	432
\tilde{b}_2	551	548	533	530
\tilde{d}_L	579	571	579	571
\tilde{d}_R	551	548	551	548
\tilde{s}_L	579	571	580	574
\tilde{s}_R	551	548	440	438
$\tilde{\tau}_1$	212	206	212	206
$\tilde{\tau}_2$	139	135	139	135
\tilde{e}_L	209	203	209	203
\tilde{e}_R	147	144	146	143
$\tilde{\nu}_e$	192	186	193	186
$\tilde{\nu}_\tau$	191	185	192	185
χ_1	104	97	103	97
χ_2	192	178	187	175
χ_3	339	356	312	327
χ_4	365	377	343	352
χ_1^\pm	191	178	186	174
χ_2^\pm	365	377	343	352
h	111	111	109	109
H	380	392	357	367
A	380	392	357	367
H^\pm	389	400	367	376

Table 2: Sparticle masses (in GeV) for the SPS1a point with $m_t = 170.9$ GeV in the R -parity conserving case (column two), and in the BNV scenario (column three). Supersymmetric masses have been calculated for first and second loop corrections in each scenario.

process, reimposing the respective boundary conditions at each iteration. We employ one-loop radiative corrections as detailed in ref. [19]. We use SPS1a mSUGRA point with $m_0 = 100$ GeV, $m_{1/2} = 250$ GeV, $A_0 = -100$ GeV, $\tan\beta = 10$ and $\text{sgn}(\mu) = +$.

By evaluating the RG equations, We have investigated the dependence of the sparticle masses on λ'' . The mass matrix for up-type squarks has no explicit dependence on λ'' ; however, it has an implicit effect due to the RG evolution. We assume that only one out

mass	1-loop $\lambda''_{ijk} = 0$	2-loop $\lambda''_{ijk} = 0$	1-loop $\lambda''_{323} = .84$	2-loop $\lambda''_{323} = .84$
\tilde{g}	628	613	624	614
\tilde{t}_1	400	399	313	313
\tilde{t}_2	594	590	574	572
\tilde{u}_L	573	565	573	565
\tilde{u}_R	552	548	552	548
\tilde{c}_L	573	565	574	568
\tilde{c}_R	552	548	552	551
\tilde{b}_1	520	514	442	440
\tilde{b}_2	551	548	530	529
\tilde{d}_L	579	571	579	571
\tilde{d}_R	551	548	551	548
\tilde{s}_L	579	571	580	574
\tilde{s}_R	551	548	447	446
$\tilde{\tau}_1$	212	207	212	207
$\tilde{\tau}_2$	139	134	138	134
\tilde{e}_L	209	203	209	203
\tilde{e}_R	147	144	146	143
$\tilde{\nu}_e$	192	186	193	186
$\tilde{\nu}_\tau$	191	185	192	185
χ_1	104	97	103	97
χ_2	193	180	189	177
χ_3	350	368	324	339
χ_4	375	388	353	362
χ_1^\pm	192	179	188	176
χ_2^\pm	375	388	352	362
h	114	114	112	112
H	391	403	368	378
A	391	403	368	378
H^\pm	400	411	377	387

Table 3: Sparticle masses (in GeV) for the SPS1a point with $m_t = 177.7$ GeV in the R -parity conserving case (column two), and in the BNV scenario (column three). Supersymmetric masses have been calculated for first and second loop corrections in each scenario.

of the set of couplings $\{\lambda''_{323}, \lambda''_{212}\}$ is non-zero at M_Z , and that only these couplings are non-zero in the running. The sparticles are not sensitive to the top mass; however we take $m_{top} = 174.3 \pm 3.4$ GeV.

In table 1 we have been calculating all sparticle masses for two different values of λ''_{ijk} ($\lambda''_{323}(M_Z) = .87$, and $\lambda''_{212}(M_Z) = .5$) up two loop corrections. Our results show the effect of λ''_{323} is quite considerable on some particles. We obtain significant shifts(25)%

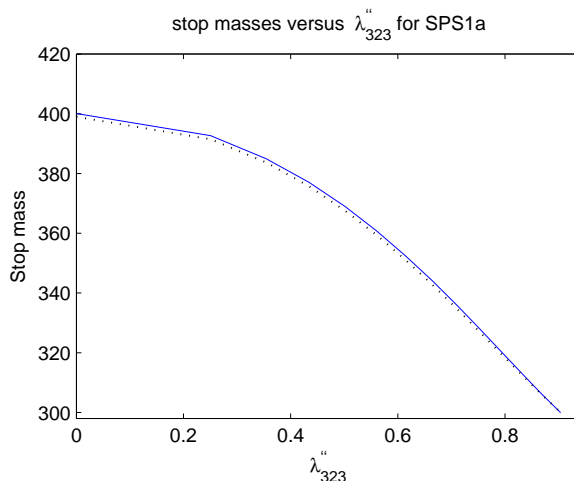


Figure 1: The light stop mass (in GeV) as a function of $\lambda''_{323}(M_Z)$ for the SPS1 Benchmark Point. Solid and dotted lines correspond to one and two -loop running respectively.

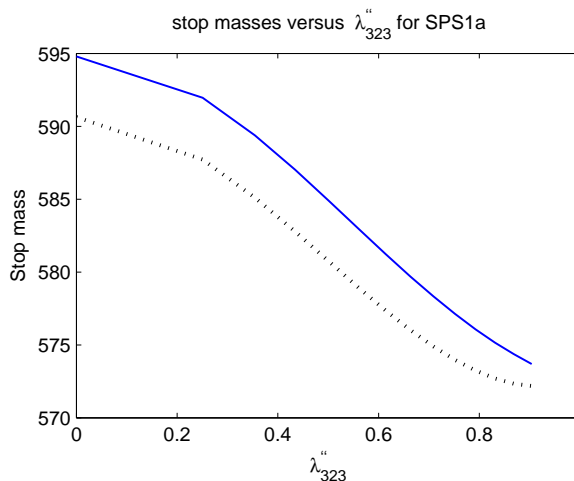


Figure 2: The heavy stop mass (in GeV) as a function of $\lambda''_{323}(M_Z)$ for the SPS1 Benchmark Point. Solid and dotted lines correspond to one and two -loop running respectively.

in the masses of $\tilde{t}_1, \tilde{b}_1, \tilde{s}_R$ which are all dominantly $SU(2)$ singlets and thus directly couple to the R -parity violating couplings; however, the corrections due to two-loop running are typically small. We also find small shifts in the corresponding doublet masses as well as the gluino mass. There are also considerable shifts (5–8%) in the masses of $\chi_3, \chi_4, H, A, H^\pm$. Moreover, we show λ''_{212} has large effect on the masses of $\tilde{s}_R, \tilde{c}_R, \tilde{d}_R$ (10%) , small effect on the gluino mass, and neglected effects on others. Our results (in the 1-loop λ''_{212} case) is typically consist with those of ref. [20]; however we have obtained smaller shifts in the masses of sparticles in compare with results of ref. [20]

In order to compare the effects of λ'' on the sparticle masses with the effects of variation of top masse on them, we have calculated sparticle masses for $m_t = 170.9$ and $m_t =$

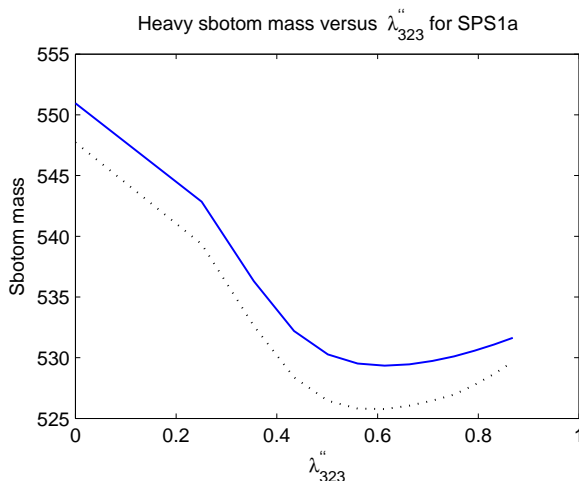


Figure 3: The heavy sbottom mass (in GeV) as a function of $\lambda''_{323}(M_Z)$ for the SPS1 Benchmark Point. Solid and dotted lines correspond to one and two -loop running respectively.

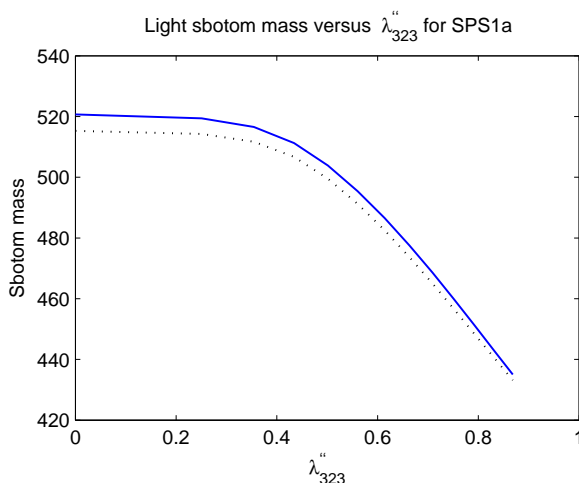


Figure 4: The light sbottom mass (in GeV) as a function of $\lambda''_{323}(M_Z)$ for the SPS1 Benchmark Point. Solid and dotted lines correspond to one and two -loop running respectively.

177.7 GeV in table 2 and 3 respectively as well as $m_t = 174.3$ GeV in table 1. Our results show the variation of top masse has neglected effects on squark and slepton masses (the squark masses have been changed ± 1 GeV when we have changed $m_t \pm 3.4$ GeV). On the other hand, the corrections due to two-loop running can be considerable for squarks.

In Figs. 1 and 2 we show the variations of the light and heavy stop masses against λ''_{323} , using first and second loop BNV β -functions for all couplings. They show the variation of the stop masses, especially the light one, with λ''_{323} is considerable; however, the effect of inclusion of two loop running is small, especially for the light one.

The variations of the light and heavy sbottom masses have been shown In Figs. 3 and 4 up two loop corrections. It is interesting that heavy sbottom mass decreases with increasing of the λ''_{323} , then it increases smoothly.

5. Conclusions

In a previous paper [21], we have studied and obtained some bounds on the λ_{ijk} and λ'_{ijk} , and presented a Form program to calculate up two-loop β -functions for R parity violating couplings; however we have not calculated explicitly the full two-loop β -functions for R parity violating MSSM couplings.

Now, we have presented the full two-loop β -functions of baryon number violating extension of the MSSM for first time. They can be used in the future analysts.

The effects of λ''_{323} and λ''_{212} on the sparticle masses have also been carried out. The results have shown significant shifts in the masses of sparticles, which are all dominantly $SU(2)$ singlets and directly couple to the λ''_{323} or λ''_{212} [20] and small shifts in the masses of others; however, The effects of λ''_{323} on the masses of some sparticles (such as the heavier neutralino and both charginos) are quite large because λ''_{323} couples to three regular third generation Yukawa couplings; therefore they obtain big shifts in their masses due to the RG evolution.

The our results have shown the sparticle masses have usually decreased against increasing of λ'' . we also have compared the effects of the first and second loop corrections. The results have shown the effect two-loop corrections in the (BNV) scenario on the sparticle masses are small, and more typically the effect is between 1 and 5%.

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